§ Puzzle of BH entropy

Classical GR $S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, R \Rightarrow R_{\mu\nu} = 0.$

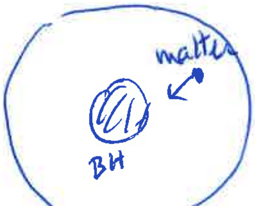
Schwarzschild: $ds^2 = -\left(1-\frac{R_H}{r}\right)dt^2 + \frac{dr^2}{1-R_H/r} + r^2 \underbrace{d\Omega_2^2}_{d\theta^2+\sin^2\theta\, d\varphi^2\ (S^2)}$

Horizon radius $R_H = 2G\underbrace{M}_{\text{mass of BH}}$

More generally, stationary BH soln. completely specified by (M, Q, J) $\Rightarrow$ BHs are very simple objects (cf. elementary particles).

Quantum theory: BHs are very complex
(Simplicity of classical limit = simplicity of averages.)
BHs are thermodynamic objects w/ T_{BH} temperature & S_{BH} entropy

Why? [Bekenstein 1973]
$\delta S_{tot} \geq 0 \Rightarrow$ BH must have entropy.



Note: $|\delta S_{BH}| \geq |\delta S_{matter}|$
$\Rightarrow$ BHs "most entropic objects".

How much? [Hawking 1975] $\longrightarrow T_{BH} = \frac{\hbar}{8\pi G M}$

$dS_{BH} = \frac{dM}{T} = 8\pi G M\, dM \Rightarrow S_{BH} = 4\pi G M^2$
$= \frac{A_H}{4G}$ ← Area of horizon

Universal law for BHs in GR * matter

$$\boxed{S_{BH} = k_B \frac{A_H c^3}{4 G \hbar} = k_B \frac{A_H}{4 l_p^2}}$$

{ Origin of holography
Wheeler

→ What are the molecules of a BH?

$$\log d(M, Q, J) \stackrel{?}{=} \frac{A_H}{4 l_p^2}(M, Q, J) + \dots$$

# of microstates

Plan of lectures

Ex ≡ Exercise
to be done by students

§ BHs in string theory

$$d(M) = \dim \mathcal{H}_{BH}(M) \overset{M/M_p\to\infty}{\sim} \exp\Big(\overset{4\pi G M^2 \ell_p^2}{\overset{\|}{S_{BH}(M)}}\Big)$$

Q: what kind of system has ↓ ?

[cf gas of particles / quantum fields. $\boxed{Ex}$ $S(M)\sim M^{3/4}\underset{\text{size}}{\underset{\uparrow}{R}}{}^{3/4}$]

[t'Hooft, Susskind, Uglum, Russo, ... 1990s]

Fundamental string  N excited oscillator modes

$$d_{st}(N) \underset{N\to\infty}{\sim} e^{\alpha\sqrt{N}} \quad \alpha>0, \quad , \quad M_{st}^2 = N/\underset{\text{fundamental const}}{\underset{\uparrow}{\ell_s^2}}$$

$$= e^{\alpha M \ell_s}$$

BH: $d\sim e^{aM^2\ell_p^2}$ | exp. growth ✓ $\ell_s \leftrightarrow \ell_p$ ✓

$d_{st}\sim e^{\alpha M\ell_s}$ | Details ✗.

More physically, when is an object a BH?

$R_H >$ char. size of system ⇒ BH

" < " " " " ⇒ planet, star, electron, ...
→ matter.. $= v_c \approx 1$

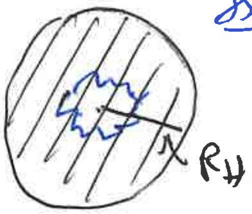
Recall that, in string theory, $S^{4d}_{eH} = \dfrac{(V_6/\ell_s^6)}{g_s^2\ell_s^2}\int d^4x\sqrt{g}\,R$

⇒ $G \approx g_s^2\ell_s^2$

⇒ $R_H = GM \approx g_s^2\ell_s^2 M$

As $g_s \searrow$, $R_H \searrow$.

$g_s \gg 1$



$d \sim e^{S_{BH}(M)}$

Macroscopic

$g_s \ll 1$

$d \sim e^{S_{micro}(M)}$

Microscopic

$R_H = l_s$ ← BH-string transition.

$$\Rightarrow g_s^2 l_s^7 M = l_s \Rightarrow g_s^2 = \frac{1}{M_s l_s} \Rightarrow S_{BH}\big|_{transition} = GM^2 = g_s^2 \frac{l_s^8 M^2}{l_s M_s} = M l_s = S_{str}!$$

More generally, Correspondence principle [Horowitz-Polchinski '96]

Set $R_{macro} = R_{micro}$

$\Rightarrow S_{macro} = S_{micro}$.

§ Breakthrough [Strominger, Vafa, Sen '95, '96]

AWSLI (3a)

2 new ingredients : (1) More charges $S(M, Q_1, \dots, P_1, P_2, \dots, J)$ (here $J = 0$)

(2) Supersymmetry (SUSY)

Idea • SUSY states $\Rightarrow M = M(\vec{Q}, \vec{P})$, independent of g_s

• As $g_s \nearrow$ "# of SUSY states" does not change

$\Rightarrow S_{micro}(\vec{Q}, \vec{P}) = S_{macro}(\vec{Q}, \vec{P})$

statistical — gravitational.

§ SUSY index

SUSY QM $\{Q, Q^\dagger\} = H$, $[H, Q] = [H, Q^\dagger] = 0$.

$(\Rightarrow H \geq 0)$

Q: Fermionic supercharge $\{(-1)^F, Q\} = 0$

Reps $H|\cdot\rangle = E|\cdot\rangle$

• $E > 0 \Rightarrow \psi = Q/\sqrt{E}$, $\psi^\dagger = Q^\dagger/\sqrt{E} \Rightarrow \{\psi, \psi^\dagger\} = 1$

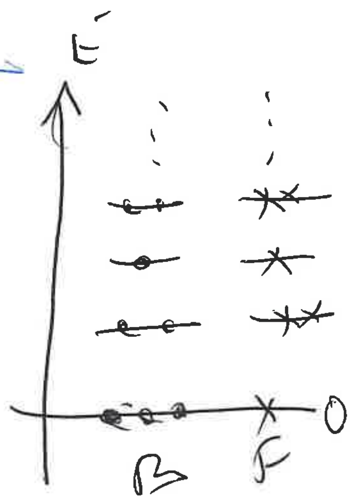
Non-BPS/long

$0 \xleftarrow{\psi} |\cdot\rangle \underset{\psi}{\overset{\psi^\dagger}{\rightleftarrows}} |x\rangle \xrightarrow{\psi^\dagger} 0$

H	E	E
$(-1)^F$	$+$	$-$.

• $E = 0$ $Q|\cdot\rangle = Q^\dagger|\cdot\rangle = 0 \Rightarrow H = 0$,

$|x\rangle$ $|x\rangle$



~~Defn~~
Witten index $\quad Z = \mathrm{Tr}\,(-1)^F e^{-\beta H}$

$$= n_0^{(B)} - n_0^{(F)}.$$

Deform $\quad H \to H_g = H + g\tilde{H}$

$$Q \to Q_g \qquad \{Q_g, Q_g^\dagger\} = H_g$$

$\Rightarrow$ spectrum shifts in pairs

$\Rightarrow \quad Z(g) = Z(0).$

- Only BPS states contribute to Z
- Z ind. of coupling const / moduli

Witten index only counts $E=0$ states

we want $E = M_{BH}$!

AWS L1 (4a)

§ BPS states & helicity supertraces in $\mathbb{R}^{1,3}$

Basic susy algebra in $\mathbb{R}^{1,3}$.

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = \gamma^\mu_{\alpha\dot\alpha} P_\mu \qquad \alpha, \dot\alpha = 1,2$$

$(\tfrac{1}{2},0)$ ↙ ↘ $(0,\tfrac{1}{2})$ spinors of $SO(1,3)$.

Reps

- Vacuum $Q_\alpha|0\rangle = \bar{Q}_{\dot\alpha}|0\rangle = 0$. susy ✓

- Massive particle $P_\mu = (m,0,0,0)$ (choose rest frame)

EX

$A_1 = Q_1,\ A_1^\dagger = \bar{Q}_{\dot 1}$, $A_2 = Q_2$, $A_2^\dagger = \bar{Q}_{\dot 2}$ ~~susy~~

$\{A_a, A_b^\dagger\} = m\,\delta_{ab} \qquad a,b = 1,2.$

γ	$\lvert\Omega\rangle$	$A_1^\dagger\lvert\Omega\rangle$	$A_2^\dagger\lvert\Omega\rangle$	$A_1^\dagger A_2^\dagger\lvert\Omega\rangle$
E	m	m	m	m
$(-1)^F$ // $(-1)^{2J_3}$	$+$	$-$	$+$	$-$
J_3	$j_3 - \frac{1}{2}$	j_3	j_3	$j_3 + \frac{1}{2}$

Tensor w/ spin $\vec{j}$ of Poincaré $\Rightarrow j - \tfrac{1}{2} \oplus \vec{j} \oplus \vec{j} \oplus j + \tfrac{1}{2}$.

Still no susy

BPS states: preserve some but not all susy

$\mathcal{N}$-extended susy

$$\{Q^I_\alpha, \bar{Q}^J_{\dot\alpha}\} = \delta^{IJ}\gamma^\mu_{\alpha\dot\alpha} P_\mu \qquad I,J = 1,2,\dots,\mathcal{N}$$

$$\{Q^I_\alpha, Q^J_\beta\} = Z^{IJ}\varepsilon_{\alpha\beta}.$$

↖ antisymmetric central charge.

$\mathcal{N}=2 \qquad Z^{IJ} = \begin{pmatrix} 0 & Z \\ -Z & 0 \end{pmatrix}$

Reps ① Vacuum : as before.

② Massive $P_\mu = (M, 0, 0, 0)$

$\boxed{EX}$ $(Q^I_\alpha, \bar{Q}^J_{\dot\alpha}) \xrightarrow[\text{combinations}]{\text{linear}} A_a, A_a^\dagger, B_a, B_a^\dagger$

$a = 1, 2.$

s.r.

$\{A_a, A_b^\dagger\} = (M+Z)\delta_{ab}$

$\{B_a, B_b^\dagger\} = (M-Z)\delta_{ab}. \qquad \Rightarrow M \geq |Z|.$

2a Short / $\frac{1}{2}$-BPS $\qquad B_a|\ \rangle = B_a^\dagger|\ \rangle = 0. \quad a=1,2$

$\Rightarrow \overline{M = Z}$

$|\Omega\rangle, \ A_1^\dagger|\Omega\rangle, \ A_2^\dagger|\Omega\rangle \ A_2^\dagger|\Omega\rangle \rightarrow$ same as ②.

2b Long / Non-BPS

$|\Omega\rangle \rightarrow$ Act by $A_a^\dagger, B_a^\dagger \Rightarrow 2^4 = 16$ dimensional

	Vac	Short	Long
$B_0 = \mathrm{Tr}\,(-1)^{2j_3}$	(± 1)	0	0
$B_1 = \mathrm{Tr}\,(-1)^{2j_3}(2j_3)$	0	0	0
$B_2 = \mathrm{Tr}\,(-1)^{2j_3}(2j_3)^2$	0	1	$0.$

$\boxed{EX}$

AdS L1
Exercise

$\boxed{\text{Ex}}$ More generally, ~~N=2,~~

$\mathcal{N}$	2	4	8
# of susys	8	16	32
∃ BPS states w/ preserve 4 susy	$\frac{1}{2}$-BPS	$\frac{1}{4}$-BPS	$\frac{1}{8}$-BPS
Break	4	12	28 susys.

4n susys unbroken ⇒ 2n pairs of zero modes (eg $A_a, A_a^\dagger$ for $\mathcal{N}=2$).

quantize zero modes ⟶

$$\boxed{B_{2n} := \frac{1}{(2n)!} \mathrm{Tr}\,(-1)^{2J_3} (2J_3)^{2n}}$$

Helicity supertrace

~~B_{2n} receives contributions from BPS.~~

Consider BPS states preserving 4 susys.

$B_{2/6/14}$ receives contributions from such BPS states
" NO " " states breaking more susy.

$\mathcal{N}=2 \to B_2$
$\mathcal{N}=4 \to B_6$
$\mathcal{N}=8 \to B_{14}$

Exercise: construct tables of reps for $\mathcal{N}=2,4,8$ & verify the above statements.

Universidad del País Vasco — Euskal Herriko Unibertsitatea
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